Closure Properties of Regular Languages

Lecture 13
Section 4.1

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Outline

Outline

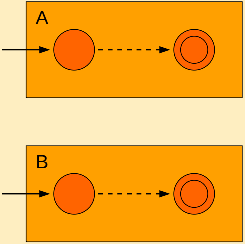
Closure

Theorem (Closure Properties of Regular Languages)

The class of regular languages is closed under the operations of complementation, union, concatenation, and Kleene star.

Closure

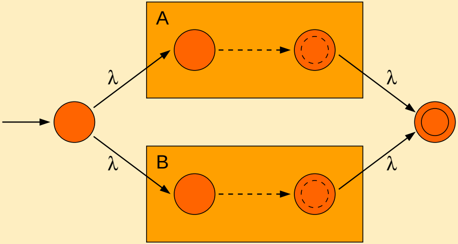
Proof for unions.



□

Closure

Proof for unions.



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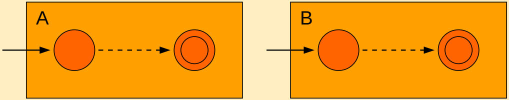
Closure

Proof for concatenations.



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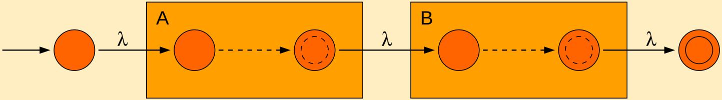
Closure

Proof for concatenations.



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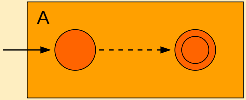
Closure

Proof for Kleene star.



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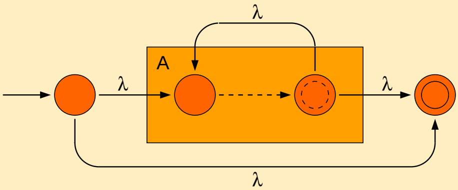
Closure

Proof for Kleene star.

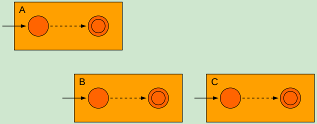


□

Closure

Example (Closure)

- A DFA for the language $(A \cup BC)^*$.



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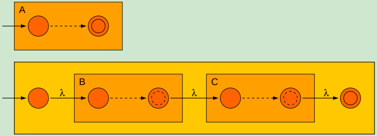

Closure

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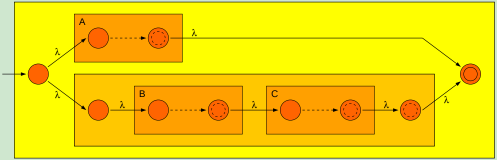

Closure

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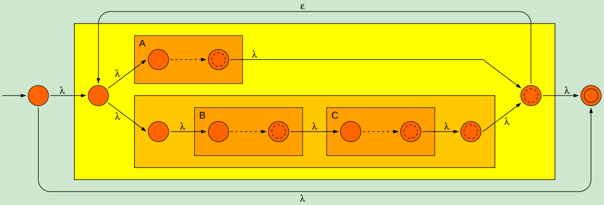
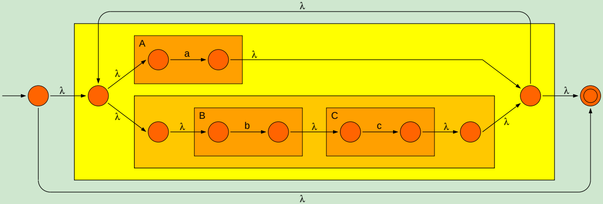


Closure

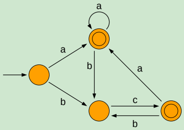
Example (Closure)

- If $A = \{\mathbf{a}\}$, $B = \{\mathbf{b}\}$, and $C = \{\mathbf{c}\}$, then we have.



Example (Closure)

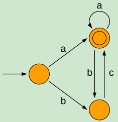
- The equivalent DFA is.



Closure

Example (Closure)

- This can be minimized to.



Outline

More Closure Properties

Corollary

The set of regular languages is closed under intersection and set difference.

Outline

Example (Intersection)

- Let $\Sigma = \{\mathbf{a}, \mathbf{b}\}$ and

$$L_1 = \{w \mid w \text{ contains } \mathbf{aba}\}$$
$$L_2 = \{w \mid w \text{ contains } \mathbf{bab}\}$$

- Design a DFA for $L_1 \cap L_2$.
- Design a DFA for $L_1 - L_2$.

Example (Intersection)

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- Design a DFA for $L_1 \cap L_2$.
- Design a DFA for $L_1 - L_2$.
- Design a DFA for $L_1 \cup L_2$.

Outline

Right Quotients

Definition (Right Quotient)

Let L_1 and L_2 be languages on an alphabet Σ. The right quotient of L_1 with L_2 is

$$L_1/L_2 = \{x \mid xy \in L_1 \text{ for some } y \in L_2\}.$$

Theorem

If L_1 and L_2 are regular languages, then L_1/L_2 is regular.

Right Quotients

Proof.

- Let $L_1 = L(M)$ and $M = (Q, \Sigma, \delta, q_0, F)$.

□

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- Let $L_1 = L(M)$ and $M = (Q, \Sigma, \delta, q_0, F)$.
- Define $M' = (Q, \Sigma, \delta, q_0, F')$ with F' defined as follows.

$\square$

Right Quotients

Proof.

- Let $L_1 = L(M)$ and $M = (Q, \Sigma, \delta, q_0, F)$.
- Define $M' = (Q, \Sigma, \delta, q_0, F')$ with F' defined as follows.
  - For each $q_i \in Q$, let $M_i = (Q, \Sigma, \delta, q_i, F)$.

$\square$

Right Quotients

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  - For each $q_i \in Q$, let $M_i = (Q, \Sigma, \delta, q_i, F)$.
  - Determine whether $L(M_i) \cap L_2 = \varnothing$.

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    - If $L(M_i) \cap L_2 \neq \varnothing$, then $q_i \in F'$.

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Proof.

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  - For each $q_i \in Q$, let $M_i = (Q, \Sigma, \delta, q_i, F)$.
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    - If $L(M_i) \cap L_2 \neq \varnothing$, then $q_i \in F'$.
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□

- The idea is that
  - x goes from q_0 to q_i for some $q_i \in Q$.
  - y goes from q_i to q_f for some $q_f \in F$ and $y \in L_2$.

Outline

Right Quotients

Example (Right Quotients)

- Let $\Sigma = \{\mathbf{a}, \mathbf{b}\}$ and

$$L_1 = L(\mathbf{aba}^*)$$
$$L_2 = L(\mathbf{b}^*\mathbf{a})$$

- That is,

$$L_1 = \{\mathbf{ab}, \mathbf{aba}, \mathbf{abaa}, \mathbf{abaaa}, \ldots\}$$
$$L_2 = \{\mathbf{a}, \mathbf{ba}, \mathbf{bba}, \mathbf{bbba}, \ldots\}$$

- Use the construction in the proof to create a DFA for L_1/L_2.

Outline

Assignment

Assignment

- Section 4.1 Exercises 1a, 2, 4, 6b, 11, 12, 14, 16, 20.
- Is the family of regular languages closed under *infinite* union? That is, if $L_1, L_2, L_3, \ldots$ are regular languages, is $L_1 \cup L_2 \cup L_3 \cup \cdots$ necessarily a regular language?
- What about infinite intersections of regular languages? Must $L_1 \cap L_2 \cap L_3 \cap \cdots$ be regular?